Rigorous Analysis of the Rectangular Waveguide Six-Port Cross Junction

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Abstract— **The modal S-matrix of the general rectangular waveguide six-port cross junction key-building block is derived by applying the mode-matching technique. Rapid convergence and accurate results are achieved by the superposition of six standing wave solutions in the common cavity regions which meet the electric wall boundary conditions in the waveguide ports rigorously. The theory is verified by measurements at a six-port cross with standard Ku-band (WR 62) rectangular waveguide ports.**

I. Introduction

THE RECTANGULAR waveguide six-port cross junction (Fig. 1) is the general key-building block for many waveguide structures of interest, such as the turnstile junction [1], E-plane or H-plane hybrid T-couplers, combined E-H T-junctions, ortho-Tee's and Purcell's junction [1], [2]. For the rigorous design of such components, it is often desirable to dispose of adequate field-theory based CAD methods which allow the fast and accurate optimization of the design parameters. As the mode-matching method in combination with the generalized S-matrix technique for suitable key-building blocks has turned out to be a very efficient design tool for many composed waveguide circuits, cf. e.g., [3], the necessary first step toward the development of an adequate CAD program comprising the related waveguide components is to derive the key-building block modal S-matrix of the waveguide six-port cross junction.

Although rectangular waveguide E- and H-plane T-junctions have already been the subject for using the mode-matching technique [3], the general rectangular waveguide six-port cross junction (Fig. 1) has not yet been investigated by this method so far. The purpose of this letter, therefore, is to present the full-wave modal S-matrix solution of this key-building block. The combination with already known key-building blocks, e.g., the double-step discontinuity [4], rectangular iris or post elements [3], or the transition from rectangular to circular waveguide [5], by the generalized S-matrix technique allows the rigorous description of adequately composed waveguide circuits. The results for the modal S-matrix of the waveguide six-port cross junction derived in this letter are verified by measurements at a Ku-band (WR 62) prototype.

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II. Theory

The full wave modal S-matrix of the waveguide six-port cross junction is derived by applying the mode-matching procedure for suitably chosen subregions (Fig. 1), like in the case of the T-junction [3].

For the waveguide subregion under consideration, the fields [3]

$$\vec{E}^{\nu} = \frac{1}{j\omega\epsilon}\nabla\times\nabla\times\vec{A}_e^{\nu} + \nabla\times\vec{A}_h^{\nu}$$
$$\vec{H}^{\nu} = -\frac{1}{j\omega\mu}\nabla\times\nabla\times\vec{A}_h^{\nu} + \nabla\times\vec{A}_e^{\nu} \qquad (1)$$

are derived from the z components of the electric and magnetic vector potentials $\vec{A}_e$, $\vec{A}_h$, respectively,

$$\vec{A}_{hz} = \sum_{i=0}^{N_h} Q_{hi}T_{hi}\left[A_{hi}e^{-\gamma_{hi}z} + B_{hi}^{+\gamma_{hi}z}\right] \qquad (2a)$$
$$\vec{A}_{ez} = \sum_{i=1}^{N_e} Q_{ei}T_{ei}\left[A_{ei}e^{-\gamma_{ei}z} - B_{ei}^{+\gamma_{ei}z}\right], \qquad (2b)$$

where A_i, B_i are the still unknown eigenmode amplitude coefficients of the forward $(-)$ and backward $(+)$ waves in z direction $\gamma_{h,e}$, are the propagation factors of the N_h and N_e considered TE_{pq} and TM_{pq} modes, respectively, and i stands for p, q. Q is a normalization factor, so that the power carried by each mode is 1 W for propagating modes, j W for evanescent TE modes, $-j$ W for evanescent TM modes [3], and T are the cross-section eigenfunctions

$$T_{hi} = N_pN_q\sqrt{Z_h}\cos\left(\frac{p\pi}{a}x\right)\cos\left(\frac{q\pi}{b}y\right) \qquad (3a)$$
$$T_{ei} = \sqrt{Y_e}\sin\left(\frac{p\pi}{a}x\right)\sin\left(\frac{q\pi}{b}y\right), \qquad (3b)$$

with

$$N_{p,q} = \frac{1}{\sqrt{1+\delta_{0p,q}}}, \qquad (4)$$

where δ is the Kronecker delta, and Z_h, Y_e are the wave impedances or admittances, respectively.

The field in the cavity subregion VII (Fig. 1) with rectangular waveguides at ports I, II, V, VI, and quadratic waveguides at ports III and IV is derived by the superposition of six suitably chosen standing wave formulations [3] $A_{h,e}^{\mathrm{VII(I)}}, A_{h,e}^{\mathrm{VII(II)}}, A_{h,e}^{\mathrm{VII(III)}}, A_{h,e}^{\mathrm{VII(IV)}}, A_{h,e}^{\mathrm{VII(V)}}, A_{h,e}^{\mathrm{VII(VI)}}$, where formulation (I) is obtained from the (1)–(3) if the boundary planes (dashed lines in Fig. 1) related to the ports II,$\cdots$,VI are

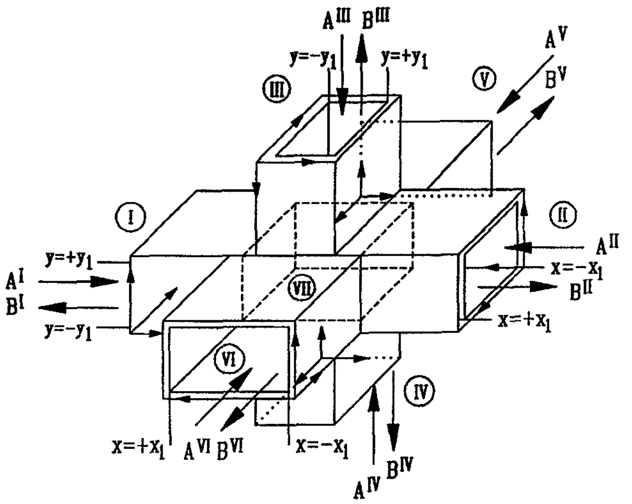

Fig. 1. Rectangular waveguide six-port cross junction.

short-circuited and port I is open; formulations (II) $\cdots$ (VI) are found analogously.

By matching the tangential electric and magnetic field components given by (1)–(3) between the empty waveguides I$\cdots$VI and the cavity region VII at the common interfaces across the boundary planes related to the ports I,$\cdots$,IV, respectively, and utilizing the orthogonal property of the modes, the still unknown amplitude coefficients can be related to each other in the form of the desired modal scattering matrix of the rectangular waveguide six-port cross junction with quadratic output waveguides at ports III, IV.

As a second step, the known modal scattering matrix of the waveguide step key-building block discontinuity [4] is connected via the generalized scattering matrix technique immediately in the boundary planes related to the ports III and IV. This yields immediately the complete modal scattering matrix of the standard six-port cross junction with rectangular waveguide port dimensions.

For the modal analysis of the rectangular waveguide six-port cross junction, sufficient asymptotic behavior has been obtained already by consideration of TE_{mn}- and TM_{mn}-modes up to $m = 8$, $n = 8$ in all waveguide sections. This is due to the fact that the electric wall boundary conditions in the six waveguide ports are met rigorously by this standing wave superposition technique.

III. Results

For the verification of the theory, the scattering parameters of a standard rectangular waveguide six-port cross junction with Ku-band waveguide (12–18 GHz) ports, i.e., WR62 waveguide housing dimensions (15.799 mm × 7.899 mm), are calculated and compared with measurements (Fig. 2 (a), (b) at a fabricated prototype.

Fig. 2 show the calculated amplitudes (solid lines) of the input reflection coefficients (Fig. 2(a)) and transmission coefficients (Fig. 2(b)), together with the measured results (symbols ∇, □, ×, ∘, Δ). Excellent agreement may be stated between the theory and the measurements.

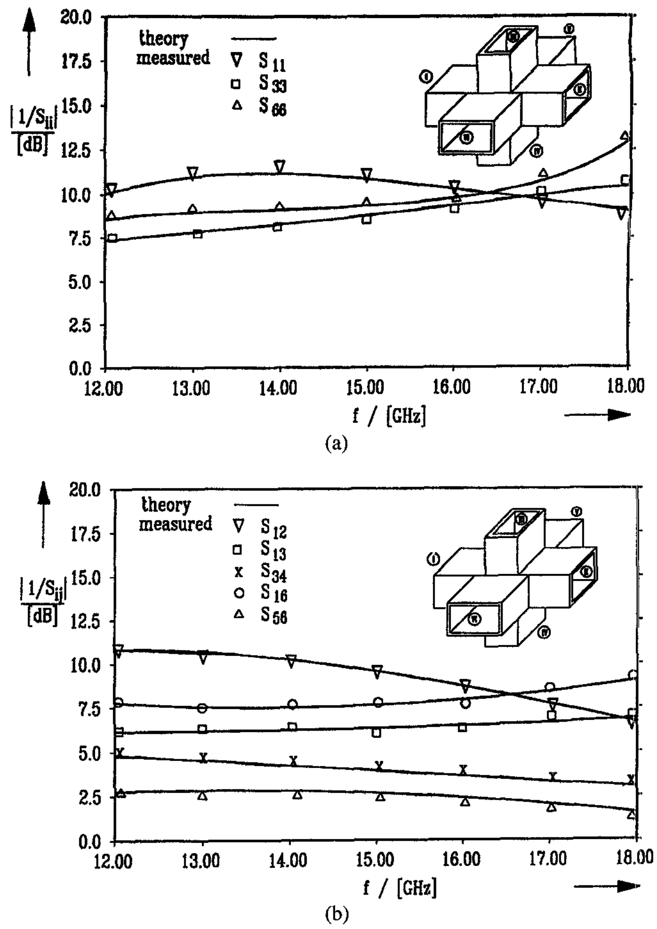

Fig. 2. Calculated (solid lines) and measured (∇, □, ×, ∘, Δ) amplitudes of the scattering parameters of a standard Ku-band rectangular waveguide six-port WR62 waveguide housing (15.799 mm × 7.899 mm). (a) Input reflection coefficients. (b) Transmission coefficients.

IV. Conclusion

The mode matching method presented in this letter achieves the accurate modeling of the general rectangular waveguide six-port cross junction. The superposition of six standing wave solutions in the common cavity region yields rapid convergence since the electric wall boundary conditions in the waveguide ports are met rigorously. Arbitrary port dimensions are included by combining the related known double-plane step discontinuity key-building blocks directly in the port planes via the generalized modal scattering matrix technique. Short-circuits at adequate planes yield directly the solution for hybrid E–H T-junctions. Moreover, by utilizing further already known key-building blocks, like the transition rectangular to circular waveguide or the waveguide bifurcation, many other structures of interest, such as the turnstile junction, or ortho-tee's may be calculated in a straight-forward manner. The theory is verified by excellent agreement with measurements.

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